1 Introduction

Logic regression proposed by Ruczinski et al. (2003) is a classification method that attempts to predict the case-control status based on Boolean combinations of binary variables. It has already been successfully applied to SNP data by Kooperberg et al. (2001).

This package contains functions that use logic regression as a wrapper to identify interesting combinations of binary variables and to measure the importance of these interactions. A description of the used methods is given in Schwender and Ickstadt (2006). Even though the intended purpose of this package is the identification of SNP interactions, it can also be applied to other types of binary (or categorical) data.

logicFS also contains a first basic Bagging version of the logic regression and a fast implementation of the Quine-McCluskey algorithm based on matrix algebra. The latter is described in Schwender (2006) and can be used to identify a minimum disjunctive normal form of a given truth table.

2 SNP Data and their Transformation into Dummy Variables

As always, the package has to be loaded.

> library(logicFS)
As example data set, we use the data set contained in `logicFS`.

```r
> data(data.logicfs)
```

`data(data.logicfs)` consists of two objects: A simulated matrix, `data.logicfs`, containing the data of 15 variables (columns) and 400 observations (rows) and a vector, `cl.logicfs`, consisting of the class labels of the 400 observations.

Each of the variables in `data.logicfs` is categorical and can take the realizations 1, 2 and 3. The class label of the first 200 observations is 1 (for case) and the class label of the remaining observations is 0 (for control). If one of the following expression is `TRUE`, then the corresponding observation is a case:

\[
\begin{align*}
\text{SNP1} & == 3 \\
\text{SNP2} & == 1 \& \text{SNP4} == 3 \\
\text{SNP3} & == 3 \& \text{SNP5} == 3 \& \text{SNP6} == 1
\end{align*}
\]

where SNP1 is in the first column of `data.logicfs`, SNP2 in the second, and so on.

Even though the variables are called SNPs, these variables have not been generated by an algorithm for simulating SNP data. So most of the values of a variable can, e.g., be 3 even though 3 should actually code the homozygous variant genotype.

Logic regression and hence the functions in `logicFS` can only handle binary predictors. So the categorical SNP variables have to be transformed into two binary dummy variables. If the homozygous reference genotype of the SNPs is coded by 1, the heterozygous genotype by 2 and the homozygous variant type by 3, then the function `make.snp.dummy` can be used to code these SNPs. Thus, a binary representation of the variables in `data.logicfs` can be obtained by

```r
> bin.snps<-make.snp.dummy(data.logicfs)
```

SNPs are coded as follows:

<table>
<thead>
<tr>
<th>SNP</th>
<th>SNP_1</th>
<th>SNP_2</th>
<th>Assumed Genotype</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>Homozygous Reference</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>1</td>
<td>Heterozygous</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>1</td>
<td>Homozygous Variant</td>
</tr>
</tbody>
</table>
3 Feature Selection Using Logic Regression

The binary dummy variables in \texttt{bin.snps} can now be used in \texttt{logic.fs} to identify potentially interesting combinations of these variables and to measure the importance of these interactions. Since the default version of the logic regression and thus the current version of \texttt{logic.fs} is based on simulated annealing, we set

\begin{verbatim}
> my.anneal<-logreg.anneal.control(start=2, end=-2, iter=10000)
\end{verbatim}

and the number \texttt{B} of considered logic regression models to 20 to speed up the computation. The feature selection is then performed by

\begin{verbatim}
> log.out<-logicFS(bin.snps, cl.logicfs, B=20, nleaves=10, rand=123, anneal.control=my.anneal)
\end{verbatim}

where we allow that a maximum of \texttt{nleaves} = 10 variables is in the logic regression models and set \texttt{rand} to 123 to make the results of this vignette reproducible.

The output of \texttt{logicFS} is given by

\begin{verbatim}
> log.out
\end{verbatim}

Selection of Interactions Using Logic Regression

Number of Iterations: 20
Sampling Method: Bagging
Logic Regression Type: Classification
Max. Number of Leaves: 10
Importance Measure: Single Tree Based On: Number of OOB Observation

The 5 Most Important Interactions:

<table>
<thead>
<tr>
<th>Importance</th>
<th>Proportion.vec.primes</th>
<th>Proportion.Freq</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>27.55</td>
<td>!X3 &amp; X8</td>
<td>0.95 !SNP2_1 &amp; SNP4_2</td>
</tr>
<tr>
<td>2</td>
<td>16.05</td>
<td>X2</td>
<td>0.95 SNP1_2</td>
</tr>
<tr>
<td>3</td>
<td>10.20</td>
<td>X6 &amp; X10 &amp; !X11</td>
<td>0.35 SNP3_2 &amp; SNP5_2 &amp; !SNP6_1</td>
</tr>
<tr>
<td>4</td>
<td>0.70</td>
<td>X10 &amp; !X11</td>
<td>0.20 SNP5_2 &amp; !SNP6_1</td>
</tr>
<tr>
<td>5</td>
<td>0.60</td>
<td>X6 &amp; X8 &amp; X10</td>
<td>0.05 SNP3_2 &amp; SNP4_2 &amp; SNP5_2</td>
</tr>
</tbody>
</table>
Not very surprisingly, only the interactions shown in Section 2 have a high import-
ance, where, e.g., \(\neg SNP2_1\) stands for NOT SNP2_1, i.e. SNP2_1 = 0. Thus, the logic expression \(\neg SNP2_1 \& SNP4_2\) means “SNP2 is of the homozy-
gous reference genotype and SNP4 is of the homozygous variant genotype.”

As displayed in Figure 1, the variable importance can be plotted by

\[
> \text{plot(log.out)}
\]

Since the names of SNPs are usually pretty long, coded names are used in the plot, where X1 codes the first variable in data, X2 the second, and so on. The original variable names are displayed in the plot if coded is set to FALSE in plot.

In the above analysis, the single tree approach of the logic regression is used. It is, however, also possible to perform a multiple tree analysis by changing the default of ntrees. Note that if this approach is applied to bin.snps one will get a lot of warnings (from glm) because of the structure of this data set (cases are unambiguously classified by the above logic expressions). With other data sets this usually will not happen.

4 Bagged Logic Regression

A first basic Bagging version of the logic regression is implemented in the function logic.bagging. Similar to the analysis with logic.fs logic.bagging is applied to bin.snps by

\[
> \text{bag.out<-logic.bagging(bin.snps, cl.logicfs, B=20, nleaves=10, rand=123, anneal.control=my.anneal)}
> \text{bag.out}
\]

Bagged Logic Regression

- **Number of Iterations:** 20
- **Sampling Method:** Bagging
- **Logic Regression Type:** Classification
- **Search Algorithm:** Simulated Annealing
- **Max. Number of Leaves:** 10
- **OOB Error Rate:** 2%

By default, both the out-of-bag error (obb=TRUE) and the importance of the variable combinations (importance=TRUE) are computed. The results of Section 3 can also be obtained by
Figure 1: Variable Importance Plot

```r
> bag.out$vim
```

**Importance Measure:** Single Tree  
**Based On:** Number of OOB Observation

**The 5 Most Important Interactions:**

<table>
<thead>
<tr>
<th>Importance</th>
<th>Proportion.vec.primes</th>
<th>Proportion.Freq</th>
<th>Expression</th>
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</thead>
<tbody>
<tr>
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<td>0.95</td>
<td>SNP2_1 &amp; SNP4_2</td>
</tr>
<tr>
<td>2</td>
<td>16.05</td>
<td>0.95</td>
<td>SNP1_2</td>
</tr>
<tr>
<td>3</td>
<td>10.20</td>
<td>0.35</td>
<td>SNP3_2 &amp; SNP5_2 &amp; SNP6_1</td>
</tr>
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<td>4</td>
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<td>0.05</td>
<td>SNP3_2 &amp; SNP4_2 &amp; SNP5_2</td>
</tr>
</tbody>
</table>

and the importance can be plotted either by `plot(bag.out$vim)` or simply by `plot(bag.out)`.  

5
The class of a new observation is predicted by majority voting and can be computed by

```r
> cl.preds <- predict(bag.out, bin.snps)
```

where we here assume that `bin.snps` contains the new observations. The misclassification rate is then computed by

```r
> mean(cl.preds != cl.logicfs)
```

[1] 0

Warning: There is currently no checking if the data set, `newbin`, containing the new observations and `data` in `logic.bagging` contain the same variables in the same order.

References


